Polytech network form for PhD Research Grants from the China Scholarship Council

This document describes the PhD subject and supervisor proposed by the French Polytech network of 14 university engineering schools. Please contact the PhD supervisor by email or Skype for further information regarding your application.

Supervisor information	
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PhD information	
Title	Robust solvers for geophysical exploration in heterogeneous media
Main topics regards to CSC list (3 topics at maximum)	I-1 Large Scale Computation, I-8 Techniques of simulation and application

Required skills in science and engineering	Scientific computing, Numerical and mathematical modelling, Programming

Subject description (two pages maximum)

Summary **:** In order to determine the make up of the earth's subsurface, sound waves are emitted from a source and their reflections from the subsurface are measured by sensors. The forward problem mapping the subsurface parameters to the observations is modelled by the elastic wave equation. Solving the inverse problem requires the repeated solution of the forward model (with many different sources), comparison of the results with the measurements, and iterative adjustment of the parameters until the discrepancy is sufficiently small. There has been recent growth of interest in working on frequency-domain problems, therefore the development of robust and performant solvers for the problems involved in each iteration of the inversion algorithm is key. This project concerns the construction, implementation, analysis and applications of fast parallel solvers for frequency-domain elastic wave-propagation in the mid- to high-frequency regime and in the presence of strong heterogeneity.

Background **:** The most fundamental model of wave propagation is the scalar wave equation $-\nabla\cdot(\kappa\nabla U)+c^{-2}\partial^2U/\partial t^2 = F$. In seismic models, this is the "acoustic approximation" of the underlying elastic wave equation, in which case $\kappa = 1/\rho$ and $c^2 = \rho c^2_p$, where ρ is density, and c_p is the speed of longitudinal waves. After Fourier transforming in time, we obtain the Helmholtz equation

$$-\nabla\cdot(\kappa\nabla u) - (\omega^2/c^2)u = f, \quad (1)$$

where ω is the frequency. Similarly, by Fourier the equations of elastodynamics, one obtains, the spatial frequency-dependent PDE

$$-Lu-\rho\omega^2u=f \quad (2)$$

In (2), f and u are vectors representing driving force and displacement, and L is the Lamé operator, whose coefficients describe the elastic properties of the medium. There is huge international interest in these problems, and much successful research has concerned good discretizations. However, the construction of efficient methods for solving the huge linear systems that subsequently arise, especially in the mid- to high-frequency regime, remains a largely open problem. One of the key challenges is to design solvers independent robust with respect to the heterogeneities in material coefficients. Thus, a successful resolution of this challenge would unlock the potential of frequency-domain formulations and produce a step change in current practice in geophysical exploration.

There is currently a large international research effort dedicated to the efficient numerical solution frequency-domain or time-harmonic PDEs, driven by the fact that in many applications, the frequency-domain formulation is a viable alternative to the time domain, provided suitably-efficient methods are available for solving the large linear systems that arise. This problem is very difficult to solve numerically. When ω is large, two difficulties arise: (i) Solutions are highly oscillatory (ii) The weak forms of these PDEs are sign-indefinite when ω is sufficiently large, and this indefiniteness is inherited by the discretised system.

Because of the features above, discretising frequency-domain wave problems leads to large indefinite linear systems. However, the fact that the systems are indefinite without a "good" preconditioner, the number of iterations grows rapidly with ω. In this context, "good" means that one wants the number of iterations to ideally be independent of ω, and for the preconditioner to be, roughly speaking, as parallelisable as possible. The ultimate goal is to solve the linear systems arising from heterogeneous wave problem in $O(N)$ time as $\omega \to \infty$, independent of the heterogeneity.

Domain decomposition (DD) methods are an attractive choice for preconditioners, since their additive versions are inherently parallel. For self-adjoint coercive scalar elliptic PDEs there is a fairly-well-developed theory for DD methods that allows very general decompositions and coarse grids, but the analysis of DD methods (and other solvers such as multigrid) for indefinite wave problems is largely an open problem. Coarse grids allow global transfer of information in the preconditioner, and increase robustness with respect to the number of the subdomains by achieving parallel scalability. The design of practical coarse spaces for frequency-domain wave problems, however, is still largely open, (partly due to the lack of a theoretical framework that allows coarse grids). One approach to obtain practical coarse spaces to extend the ideas from [1] and [2] to the case of elastic waves, or alternatively design more sophisticated coarse spaces more adapted to heterogeneous problems.

<u>Aims and objectives</u>

The most important research question for this project would be: how the coarse spaces already applied successfully to Helmholtz equations can be generalised efficiently to the elastic wave equations? We will address this question by analysing mathematically different theoretical strategies by validating them numerically on relevant test cases. The proposed plan of work includes

• Generalisation of two-level preconditioners in the spirit of [1] and [2] already developed for the Helmholtz equations;

• Analyse the influence of absorption on the preconditioner.

• Numerical assessment on heterogeneous benchmark test cases as in [1].

[1] M. Bonazzoli, V. Dolean, I. G. Graham, E. A. Spence, and P.-H. Tournier. Domain decomposition preconditioning for the high-frequency time-harmonic Maxwell equations with absorption. submitted for publication - add arxiv listing, 2017.

[2] C. Farhat, A. Macedo, and M. Lesoinne. A two-level domain decomposition method for the iterative solution of high frequency exterior Helmholtz problems. Numerische Mathematik, 85:283–308, 2000.